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REGULAR ARTICLE

# A quadratic programming model for the optimization of off-gas networks in integrated steelworks

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**Abstract.** The European steel industry is constantly promoting developments, which can increase efficiency and lower the environmental impact of the steel production processes. In particular, a strong focus refers to the minimization of the energy consumption. This paper presents part of the work of the research project entitled “Optimization of the management of the process gas network within the integrated steelworks” (GASNET), which aims at developing a decision support system supporting energy managers and other concerned technical personnel in the implementation of an optimized off-gases management and exploitation considering environmental and economic objectives. A mathematical model of the network as a capacitated digraph with costs on arcs is proposed and an optimization problem is formulated. The objective of the optimization consists in minimizing the wastes of process gases and maximizing the incomes. Several production constraints need to be accounted. In particular, different types of gases are mixing in the same network. The constraints that model the mixing make the problem computationally difficult: it is a non-convex quadratically constrained quadratic program (QCQP). Two formulations of the problem are presented: the first one is a minimum cost flow problem, which is a linear program and is thus computationally fast to solve, but suitable only for a single gas network. The second formulation is a quadratically constrained quadratic program, which is slower, but covers more general cases, such as the ones, which are characterized by the interaction among multiple gas networks. A user-friendly graphical interface has been developed and tests over existing plant networks are performed and analyzed.

## 1 Introduction

Steel production is among the largest energy-intensive industrial processes in the world, as well as one of the most important CO<sub>2</sub> emission sources. In particular, in the integrated steelworks, the blast furnace (BF) and the basic oxygen furnace (BOF) need 13–14 GJ/t of produced steel [1]. In addition, steel industry is responsible for about 4–5% of total world CO<sub>2</sub> emissions, as it is deeply dependent on fossil fuels [2,3]. Furthermore, energy cost represents about 20% of the total operation cost [4].

However, the major role of steel utilization in the modern society is undeniable. In order to improve energy saving and to reduce environmental impact and total operation cost, the optimal management of the energy resources produced inside the plant, such as off-gases, becomes decisive. The gases produced in the integrated steelworks contain significant amount of carbon monoxide and hydrogen and, for this reason, they represent a good

replacement of natural gas (NG) in all the operation where heat, steam or electricity are required or need to be produced.

Their use inside the steelmaking facilities is not new: they are directly reused in several unit operations for heating purposes or are exploited in boilers and in power plants, respectively, for steam and electricity productions. However, no optimization exists for such reuse and situations of overproduction or underproduction of gas can often occur. In the first case, the gasholders where the gas is stored become full and the excess gas is flared or the production is slowed down, in order to allow the decrease of the gasholder level. In the second case, NG is purchased in order to satisfy the demand of the different form of energy inside the plant. Consequently, these situations correspond to economic losses and increase of environmental impact.

The off-gases produced during steel production are valuable by-products. Therefore, in literature different works can be found, which deals with the maximization of the use of such gases. Some of them refer to the increase of their reuse by producing chemical products; a complete review on this topic is provided in [5]. More specific studies can be found in [6] and in [7], respectively, for methanol

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production by utilizing latent heat and endothermic heat of reaction from converter gas and for biomethanation of BF gas (BFG) through the use of anaerobic granular sludge and exogenous hydrogen addition. The reuse of some of the produced steelmaking off-gases in a cogeneration process is presented in [8] and a significant reduction of environmental impact is proved through a life cycle analysis.

Other research works are focused on the development of effective models and strategies for process optimization. Indeed, these works are related on the development of tools that are able to improve the internal management of the off-gases. An exemplar work was carried out by Porzio et al. and is described in [9–12]. Such work aimed at offline monitoring (and reduction) of CO<sub>2</sub> emissions and optimization of the distribution of the off-gases by exploiting a decision support system (DSS), including advanced modelling and multi-objective optimization techniques. Starting from the results obtained in the previously cited works, a new and improved DSS is under development during the European project entitled “Optimization of the Management of the Process Gas Network within the Integrated Steelworks” (GASNET). The main reason for the development of a novel DSS lies in the need to provide online and more accurate management of off-gases. The previous tools consider only a limited number of constraints, neglect some gas users (e.g. power plants) and do not dynamically forecast the off-gases (and related energy carriers) production and request. The new DSS is composed of different units:

- a model library, which is able to forecast the production and the demand of off-gases and related energy carriers by the different producers or users in the whole steelmaking plant, as for instance described in [13–16];
- an optimization tool, containing different optimization techniques in order to offline and online optimize the distribution of off-gases in steelworks;
- a graphical user interface in order to easily visualize the results of the different units of the DSS also in the form of key performance indicators [17,18].

The work presented in this paper is devoted to the development of a model for optimal exploitation of energy resources in integrated steelworks through application of non-linear optimisation techniques and it is part of the optimization tool of the DSS previously described.

The paper is structured as follows: the mathematical model is developed in Sections 1 and 2; Section 3 presents a test case by also showing the graphical interface of the developed software that applies the model; finally, some concluding remarks are provided in Section 4.

## 2 The model

### 2.1 Gas networks in integrated steelworks

During iron- and steel-making processes three kinds of off-gases can be produced: BFG, basic oxygen furnace gas (BOFG) and coke oven gas (COG).

In this work, the gas network of a real integrated steelwork is modelled. This gas network is composed only of a BF network and a BOF network, as coke is not produced inside the considered steelwork. The BFG, which is

produced by one or more BFs, flows along the pipe network and can be stored into gasholders, burnt by flares or sent to gas consumers such as power plants, hot blast stoves, thermal oil heaters, or pulverized coal injection (PCI) plants. Similarly, in the BOF network, basic oxygen furnaces produce the gas and it can be stored, burnt or sent to consumers. Moreover, external natural gas can be injected into the network to satisfy the consumers’ needs.

The gas and steam networks are dynamically connected, with the steam network acting as a gas consumer. In this work, the entire steam network is modelled as a single consumer process, by neglecting the related dynamics.

The overall steam and gas network system is modelled as a digraph, namely a set of vertices connected by directed arcs. The vertices represent the producer processes, the consumer processes and the joints where two or more pipes meet. The arcs are the pipes connecting the different vertices and they are oriented according to the gas flow direction. A capacitated digraph network is defined over this digraph and a (gas) flow meeting the requirements is sought: a demand (or balance) is defined for every vertex. This balance equals the difference between the amount of gas flowing in a vertex and the amount of gas flowing out of the vertex. Every arc (pipe) has a lower and upper capacity, standing for the minimum and maximum flow allowed on that pipe. In addition, a number of constraints and an objective function are defined.

The previously described mathematical structure is commonly used for optimization models. In [19,20], more details on the digraphs and the theory of network flows are provided.

The producer processes have a given supply, or negative demand (the amount of gas/energy that is produced). The consumers have a positive demand (the required amount of gas that must be sent to the consumer). A gasholder  $h$  is a vertex whose demand depends on the current filling level and lies in the range  $[GHLmin(h), GHLmax(h)]$ , where  $GHLmin(h)$  and  $GHLmax(h)$  are, respectively, the minimum and maximum possible filling level for the gasholder  $h$ .

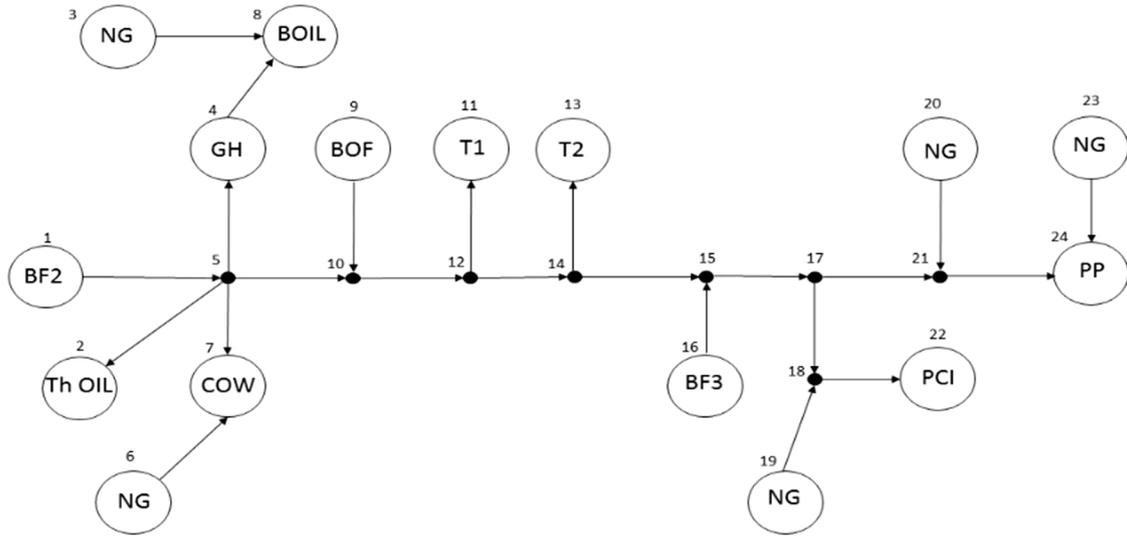
Note that both producers and consumers have an energy demand, while the gasholder demand (filling level) refers to the amount of gas (expressed in Nm<sup>3</sup>) it can contain.

When a mixture of more gases is flowing into the same pipe, the energy of the mixture is calculated as  $\sum_g x_g \cdot LHV(g)$  where  $x_g$  is the amount of gas  $g$  expressed in Nm<sup>3</sup> that is flowing and  $LHV(g)$  is the lower heating value of gas  $g$ .

For each vertex, an energetic balance must be formulated:

- flares and consumers have a non-negative balance;
- natural gas providers and other producers must have a non-negative balance;
- gasholders balance must respect the minimum and maximum range;
- all connection nodes must have zero balances (all the gas that flows in must flow out).

There are volumetric lower and upper bounds on the amount of gas that must flow in every pipe.



**Fig. 1.** The network digraph.

A cost function is defined by taking into account the revenues obtained from selling off-gas to consumers (and the costs of burning gas into flares or buying from natural gas providers and the cost of structural changes in the network like build or demolition of pipes.

This network structure with its constraints and cost function can be formulated as an optimization problem that falls under the category of quadratic constrained quadratic programming (QCQP), which is formalized in the following section.

## 2.2 QCQP model

Let  $x_{ij}^g$  be the amount of gas  $g$  in  $\text{Nm}^3$  that is flowing through the pipe starting from node  $i$  and connecting it with node  $j$  in a given time step. The equation for the energetic balance of producers is the following:

$$\sum_{g,j} (x_{pj}^g - x_{jp}^g) \cdot LHV(g) = Eb(p), \quad (1)$$

where  $Eb(p)$  is the (negative) energy demand for producer  $p$ .

Similarly, the equation for the energetic balance of consumers is as follows:

$$\sum_{g,i} (x_{ci}^g - x_{ic}^g) \cdot LHV(g) = Eb(c), \quad (2)$$

where  $Eb(c)$  is the (positive) energetic demand of consumer  $d$ .

Flares must have a non-negative balance, while natural gas sources must have a non-positive balance. Therefore, the following inequalities hold for every flare  $f$  and every natural gas provider  $n$ :

$$\sum_{g,j} (x_{fj}^g - x_{jf}^g) \geq 0, \quad (3)$$

$$\sum_{g,j} (x_{nj}^g - x_{jn}^g) \leq 0. \quad (4)$$

Gasholders  $h$  have a current filling level  $GHL(h)$  and a minimum and maximum filling level  $GHLmin(h)$ ,  $GHLmax(h)$ , respectively. Therefore, the balance at every gasholder  $h$  is as follows:

$$\begin{aligned} GHLmin(h) - GHL(h) &\leq \sum_{g,j} (x_{hj}^g - x_{jh}^g) \\ &\leq GHLmax(h) \\ &\quad - GHL(h). \end{aligned} \quad (5)$$

Moreover, there are lower and upper capacities  $l_{ij}$ ,  $u_{ij}$  for each pipe from  $i$  to  $j$ . Therefore,  $l_{ij} \leq x_{ij}^g \leq u_{ij}$  for every  $g$ ,  $i, j$  such that there is a pipe from  $i$  to  $j$ , and  $x_{ij}^g = 0$  for every  $g$  and every  $i, j$  such that there is no pipe from  $i$  to  $j$ .

Given that different type of gases are possibly mixing into the same pipes, the concentration of gas  $\bar{g}$  in the mixed flow entering a node  $\bar{i}$  must be the same concentration that is flowing out of  $\bar{i}$  on every pipe  $ij$ . Therefore, for every  $\bar{i}, j, \bar{g}$ :

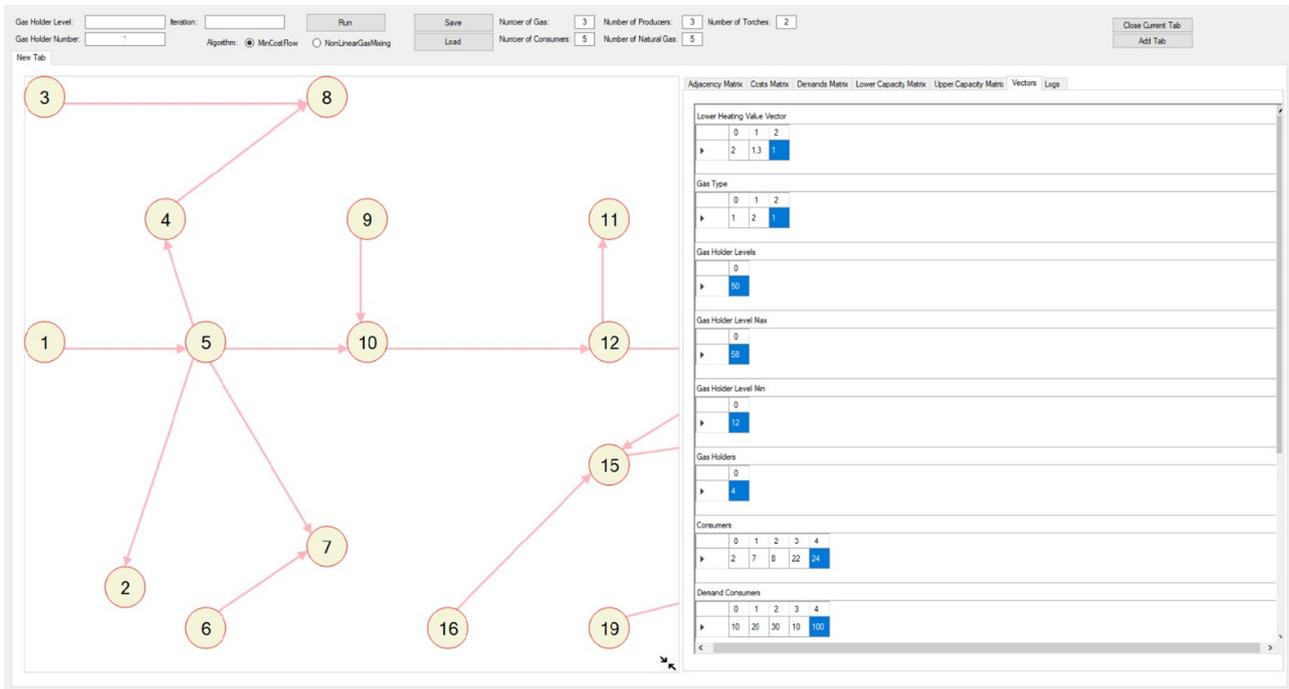
$$\sum_j x_{\bar{j}\bar{i}}^{\bar{g}} \cdot \sum_g x_{\bar{j}\bar{i}}^g = x_{\bar{j}\bar{i}}^{\bar{g}} \cdot \sum_{j,g} x_{\bar{j}\bar{i}}^g. \quad (6)$$

These equations introduce non-linearity in the constraints; more precisely, the constraints are quadratic and non-convex. This inevitably slows down the computation of an optimal solution.

Let  $P_{ng}$  be the price of purchasing one  $\text{Nm}^3$  of natural gas. Let  $P_f$  be the price of burning one  $\text{Nm}^3$  of off-gas into a flare  $f$  and let  $P_{Ec}$  be the revenue from selling one  $MJ$  of energy to consumer  $c$ .

The function to minimize is the sum of all costs minus the revenues:

$$\begin{aligned} F(x) &= \sum_{n \in NG, g} x_{ni}^g \cdot P_{ng} + \sum_{f \in TF, j, g} x_{jf}^g \cdot P_f \\ &\quad - \sum_{c \in C, j, g} x_{jc}^g \cdot LHV(g) \cdot P_{Ec}. \end{aligned} \quad (7)$$

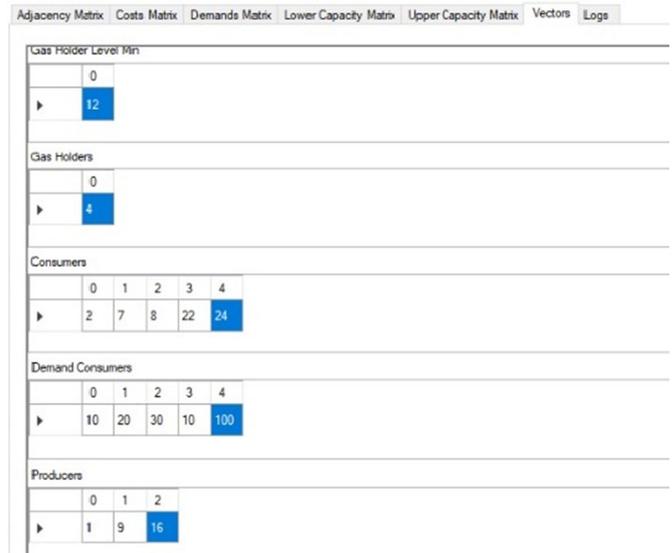


**Fig. 2.** Graphic interface: in the left part, it is possible to build the digraph part, in the top and right part the parameters of the problem are specified.

The problem of minimizing  $F(x)$  subject to all the above constraints is a non-convex QCQP, which belongs to the category of NP-complete problems: it is computationally hard to find an optimal solution. There is a subcase of this general scenario, which is much faster to solve corresponding to the situation in which one type of gas is flowing into the network. In this case, all the model equations are kept, with  $g$  being only one element, except equation (6), which is discarded. This problem is a minimum cost flow problem, which can be solved in polynomial time, namely a solution can be found even for very large networks in a reasonable amount of time.

In order to have a model that accounts for the possibility of pipes construction/demolition, activation variables  $y_{ij}$  are added to the model: for every possible connection  $ij$ ,  $y_{ij} = 0$  if the pipe  $ij$  is not activated and  $y_{ij} = 1$  otherwise. Therefore, the upper and lower bounds constraints are extended to all possible connections  $ij$ : the constraints  $y_{ij} \cdot l_{ij} \leq x_{ij}^g \leq y_{ij} \cdot u_{ij}$  define the activation variables, while ensuring that the flow stays within the pipe capacities.

There are three types of cost: a maintenance cost, a demolition cost (for pipes already in the original network) and a building cost (for pipes not in the original network). These costs are assumed to be depreciated over the used time unit. The cost  $c_{ij}$  associated with the activation variables is defined as follows: if a pipe  $ij$  is present in the original network,  $c_{ij}$  is the difference between maintenance and demolition cost for that pipe; if the pipe  $ij$  is not in the network,  $c_{ij}$  is the sum of maintenance and building cost.



**Fig. 3.** A zoom on the right area of the interface.

The component that must be added to the objective function is the sum over all possible connections  $ij$  of  $y_{ij} \cdot c_{ij}$ . Therefore, the objective function becomes as follows:

$$G(x, y) = F(x) + \sum_{i,j} y_{ij} \cdot c_{ij}. \quad (8)$$

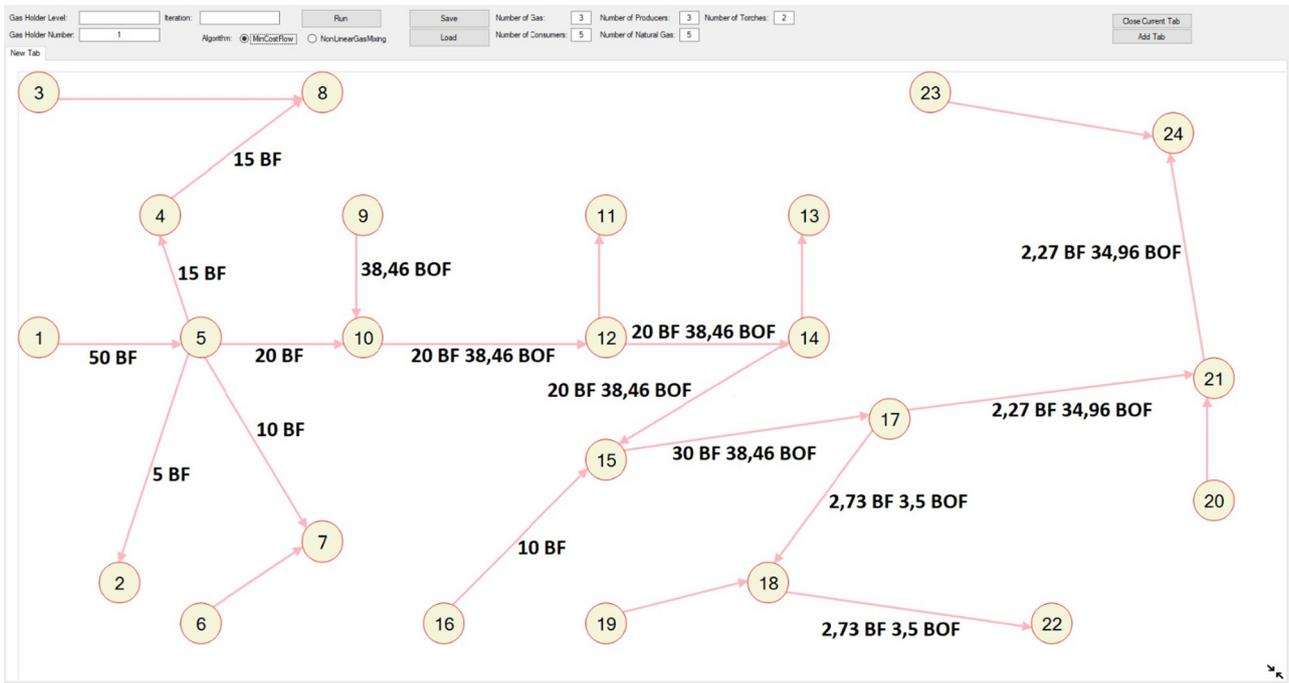


Fig. 4. Optimal solution. The displayed gas quantities are intended in  $\text{Nm}^3$  per time unit.

### 3 A test case

A real gas network has been modelled. Figure 1 shows the digraph representing the network. The whole BOF network has been modeled as a single producer node (Node 9); other producers are BF2 and BF3 (Nodes 1 and 16) and five NG providers (Nodes 3, 6, 19, 20, 23). On the consumers side, there is one node for the steam network, called BOIL (Node 8), one for the hot blast stoves (Node 7) and one for the thermal oil heaters (Node 2); other consumers are PCI (Node 22), a power plant (Node 24) and two flares (Nodes 11 and 13). Finally, there is a BF gasholder (Node 4).

A graphic interface has been designed in order to provide all the parameters. The appearance of such interface is depicted in Figures 2 and 3. For confidentiality constraints, the parameters indicated in the figure (such as gasholders filling level or energy demands) are not real: they are fictitious data of the same order of magnitude.

Through this interface, it is possible to manually build the network digraph and insert the needed values such as node numbers of consumers, producers, flares, gasholders and natural gas providers. Moreover, energetic demands, costs and revenues for the objective functions can be added.

In the test case, the algorithm finds an optimal solution with a time in the order of five minutes. The interface displays the solution such as exemplarily depicted in Figure 3. The amount of each gas type flowing in a pipe is displayed on the corresponding arc. For instance,  $27.27\text{Nm}^3$  of BF gas and  $34.96\text{Nm}^3$  of BOF gas are flowing from Node 21 to the power plant (Node 24) (Fig. 4).

### 4 Conclusion

In this work, a framework for the strategy and management optimization of off-gas systems in integrated steelworks has been presented. Graph theory allows modelling a complex system like that of the interaction between gas and steam networks with considerable advantages, from the point of view of modularity and the possibility of finding optimal solutions for the distribution of gases between producers and consumers. The developed model also allows to study the offline optimization of the gas network architecture, through the analysis of the effects of possible new connections between producers and consumers. The developed approach has been successfully applied to a real integrated steelwork gas-steam system, in which the interaction between BFG, BOFG and steam networks have been studied and optimized. Finally, a graphical user interface has been developed in order to simplify the management of off-gas distribution, as a module of a more complex DSS developed during the GASNET project.

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## References

1. International Energy Agency, Energy technologies perspective 2010, Scenarios and strategies to 2050, 2010, Available from [http://www.oecd-ilibrary.org/energy/energy-technology-perspectives-2010\\_energy\\_tech-2010-en](http://www.oecd-ilibrary.org/energy/energy-technology-perspectives-2010_energy_tech-2010-en), [last accessed Jan. 2019]
2. K. Baumert, T. Herzog, J. Pershing, Navigating the numbers: Greenhouse gas data and international climate policy, World Resources Institute report, Washington, 2005, ISBN: 1-56973-599-9, Available from [www.oecd.org/dataoecd/28/43/36448807.pdf](http://www.oecd.org/dataoecd/28/43/36448807.pdf), [last accessed Jan. 2019]
3. Worldsteel, Steel's contribution to a low carbon future: Worldsteel position paper, 2017, Available from [worldsteel.org/publications/position-papers/Steel-s-contribution-to-a-low-carbon-future.html](http://worldsteel.org/publications/position-papers/Steel-s-contribution-to-a-low-carbon-future.html), [last accessed: Jan. 2019]
4. H. Kong, E. Qi, H. Li, G. Li, X. Zhang, An MILP model for optimization of by-product gases in the integrated iron and steel plant, *Appl. Energy* **87**, 2156–2163 (2009)
5. W. Uribe-Soto, J.-F. Portha, J.-M. Commenge, L. Falk, A review of thermochemical processes and technologies to use steelworks off-gases, *Renew. Sustain. Energy Rev.* **74**, 809–823 (2017)
6. N. Maruoka, T. Akiyama, Exergy recovery from steelmaking off-gas by latent heat storage for methanol production, *Energy* **31**(10–11), 1632–1642 (2006)
7. Y. Wang, C. Yin, M. Tan, K. Shimizu, Z. Lei, Z. Zhang, I. Sumi, Y. Yao, Y. Mogi, Biomethanation of blast furnace gas using anaerobic granular sludge via addition of hydrogen, *RSC Adv.* **8**(46), 26399–26406 (2018)
8. S.G. Garcia, V.R. Montequin, R.L. Fernández, F. Ortega-Fernández, Evaluation of the synergies in cogeneration with steel waste gases based on Life Cycle Assessment: A combined coke oven and steelmaking gas case study, *J. Clean. Prod.* **217**, 576–583 (2019)
9. G.F. Porzio, B. Fornai, A. Amato, N. Matarese, M. Vannucci, L. Chiappelli, V. Colla, Reducing the energy consumption and CO<sub>2</sub> emissions of energy intensive industries through decision support systems—An example of application to the steel industry, *Appl. Energy* **112**, 818–833 (2013)
10. G.F. Porzio, V. Colla, N. Matarese, G. Nastasi, T.A. Branca, A. Amato, B. Fornai, V. Marco, M. Bergamasco, Process integration in energy and carbon intensive industries: An example of exploitation of optimization techniques and decision support, *Appl. Therm. Eng.* **70**(2), 1148–1155 (2014)
11. G.F. Porzio, G. Nastasi, V. Colla, M. Vannucci, T.A. Branca, Comparison of multi-objective optimization techniques applied to off-gas management within an integrated steelwork, *Appl. Energy* **136**, 1085–1097 (2014)
12. A. Maddaloni, G. Porzio, G. Nastasi, V. Colla, T.A. Branca, Multi-objective optimization applied to retrofit analysis: A case study for the iron and steel industry, *Appl. Therm. Eng.* **91**, 638–646 (2015)
13. I. Matino, S. Dettori, V. Colla, V. Weber, S. Salame, Application of Echo State Neural Networks to forecast blast furnace gas production: Pave the way to off-gas optimized management, *Energy Procedia* **158**, 4037–4042 (2019)
14. I. Matino, S. Dettori, V. Colla, V. Weber, S. Salame, Two innovative modelling approaches in order to forecast consumption of blast furnace gas by hot blast stoves, *Energy Procedia* **158**, 4043–4048 (2019)
15. S. Dettori, I. Matino, V. Colla, V. Weber, S. Salame, Neural Network-based modeling methodologies for energy transformation equipment in integrated steelworks processes, *Energy Procedia* **158**, 4061–4066 (2019)
16. I. Matino, S. Dettori, V. Colla, N. Zapata, S. Bastida, J. Bartel, Forecasting of basic oxygen furnace gas production through Echo State Neural Networks, in: 8th European Oxygen Steelmaking Conference—EOSC 2018, Taranto, Italy, 2018
17. V. Colla, I. Matino, S. Dettori, A. Petrucciani, A. Zaccara, V. Weber, S. Salame, N. Zapata, S. Bastida, A. Wolff, R. Speets, L. Romaniello, Assessing the efficiency of the off-gas network management in integrated steelwork, *Matériaux et Techniques* **107**(1), 104 (2019)
18. V. Colla, I. Matino, S. Dettori, A. Zaccara, A. Petrucciani, V. Weber, S. Salame, N. Zapata, S. Bastida, A. Wolff, R. Speets, L. Romaniello, A way to quantify the efficiency of the gas network management in integrated steelworks, in: 4th European Conference on Clean Technologies in the Steel Industry—CLEANTECH 4, Bergamo, Italy, 2018
19. J. Bang-Jensen, G. Gutin, *Digraphs: Theory, algorithms and applications*, 2nd Edition, Springer-Verlag, London, 2009
20. R. Ahuja, T. Magnanti, J. Orlin, *Network flows*, Prentice Hall, Upper Saddle River, New Jersey, 1993

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